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Risk Analysis of Cascading Disturbances among Power System Players

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SUMMARY

Cascading disturbances have recently occurred in major power systems around the world. Multiple players in deregulated markets must share the associated risks. This paper describes a formal method to evaluate power system risk and performance indexes, and the contribution and cross impact of several agents and market players during system outages. The method is based on the propagation of cascading disturbances through the power grid topology. Besides intrinsic forced and programmed outages, other issues are treated like security and dependability of protection, breaker failures, and restoration or automatic reclosing after a disturbance. Their joint contribution determines the overall level of continuity at each network node, and liability of each player. The models are based on reachability matrices for cascading outages, adequate to avail the impact of disturbances, maintenance, operating and planning actions from each asset owner and player. The resultant matrix model can be used as a grid failure simulator, to estimate not only continuity and risk indexes, but also statistics like Expected Energy and Demand Not Supplied (EENS, EDNS) by each player, and Loss of Load Probability and Expectation (LOLP, LOLE) of each agent or consumer.

KEYWORDS

Risk, Power Quality, Performance.

1. Introduction

Cascading disturbances in deregulated electric power systems have raised concern and doubts about responsibilities for operational risks. Continuity of power delivery to final consumers is now the result of a distributed effort of several players. Disturbances can reach far away from its original cause, affecting distant markets and several grid agents. Failures on customer installations can also impact the continuity of other network nodes, raising questions of customer responsibilities beyond their own load.

Most traditional performance indexes like System Average Interruption Frequency and Duration (SAIFI, SAIDI, etc.) evaluate power quality as related to continuity, but are insufficient to ascertain the liability of various players, including customers, generation, transmission and distribution companies. Being symptoms of inadequacies, they do not point to their causes, or who should be charged. Reference values, based on historical data, linked to economic penalties for not attaining desired levels, serve as incentive to reduce system risks and power quality, but do not signal where corrections are needed.

The first part of this paper develops a vector representation of forced and programmed outage of items in a power grid, including breaker and protection reliability and dependability, connected loads and outage duration, partitioned among asset owners. A small, multi-company power system is used, throughout the paper, to illustrate the approach. The second part introduces adjacency and reachability matrices to model the propagation of outage events on the power system grid. Graph theoretic concepts are used to support these models. The third part uses these matrices to calculate the expected overall outage frequency and duration of each grid point, partitioned according to forced and programmed outages, due to equipment, protection and breaker failures. Contribution of each company to these metrics is evaluated as a measure of their cross impact and risk sharing. The fifth part, filters the model to assess the frequency and duration of outages of selected points in the grid, like the border busses or feeders among transmission and distribution systems, regions, generators and consumers, following regulatory rules, like minimum interruption duration (typically from 1 to 3 minutes). The final part extends the model to estimate other indicators of power system risks, such as Expected Energy and Demand Not Supplied (EENS, EDNS), and Loss of Load Probability and Expectation (LOLP, LOLE) for each grid point and agent, partitioned among asset owners, appraising their liability. The conclusions summarize the features of the model, to avail the risk sharing and cross impact of cascading disturbances among grid players.

2. Simbology

$\mathbf{A}_{\mathrm{I}}$ – Forced Reachability Matrix
$\mathbf{A}_{\mathrm{I}ij}$ – Forced Reachability from Player *i* to *j*
$\mathbf{A}_{\mathrm{P}}$ – Planned Reachability Matrix
$\mathbf{A}_{\mathrm{P}ij}$ – Planned Reachability from Player *i* to *j*
C – Intrinsic Protection Reliability Vector
$\mathbf{C}_{\mathrm{D}}$ – Diagonal Matrix of **C**
D – Mean Outage Duration Vector
$\mathbf{D}_{\mathrm{A}}$ – Total Outage Duration Vector
$\mathbf{D}_{\mathrm{AC}}$ – Total Outage Duration due to Protection Failure
$\mathbf{D}_{\mathrm{AI}}$ – Total Forced Outage Duration Vector
$\mathbf{D}_{\mathrm{A}ij}$ - Total Forced Outage Duration of Player *j* due to *i*
$\mathbf{D}_{\mathrm{AP}}$ – Total Planned Outage Duration Vector
$\mathbf{d}_{\mathrm{I}}$ – Intrinsic Forced Outage Duration Vector
$\mathbf{d}_{\mathrm{ID}}$ – Diagonal Matrix of $\mathbf{d}_{\mathrm{I}}$
DIPC – Vector of Outage Duration of Control Points
DIPCM – Mean Outage Duration of Control Points
DMIPC – Maximum Outage Duration of Control Points
EDNS – Vector of Expected Demand Not Supplied
DNS – Vector of Mean Demand Not Supplied
$\mathbf{DNS}_{A}$ – Vector of Total Demand Not Supplied
$\mathbf{DNS}_{I}$ – Demand Not Supplied from Forced Outages
$\mathbf{DNS}_{P}$ - Demand Not Supplied from Planned Outages
DNS_T – Total Demand Not Supplied
$\mathbf{d}_{\mathrm{P}}$ – Intrinsic Planned Outage Duration Vector
$\mathbf{d}_{\mathrm{PD}}$ –Diagonal Matrix of $\mathbf{d}_{\mathrm{P}}$
EENS – Vector of Expected Mean Energy Not Supplied
ENS – Vector of Mean Energy Not Supplied
$\mathbf{ENS}_{\mathrm{A}}$ – Vector of Total Energy Not Supplied
$\mathbf{ENS}_{\mathrm{I}}$ – Energy Not Supplied from Forced Outages
$\mathbf{ENS}_{P}$ - Energy Not Supplied from Planned Outages

F – Vector of Total Outage Frequency
$\mathbf{F}_C$ **-** Total Outage Frequency from Protection Failure
$\mathbf{F}_{Cij}$ – Contribution Vector of $\mathbf{F}_C$ from Player *i* to *j*
$\mathbf{F}_D$ –Diagonal Matrix of **F**
$\mathbf{f}_I$ – Intrinsic Forced Outage Frequency Vector
$\mathbf{f}_{ID}$ –Diagonal Matrix of $\mathbf{f}_I$
$\mathbf{F}_I$ – Total Forced Outage Frequency Vector
$\mathbf{F}_{Iij}$ – Contribution Vector of $\mathbf{F}_I$ from Player *i* to *j*
$\mathbf{F}_{Ij}$ – Sub Vector of $\mathbf{f}_I$ from Player *j*
$\mathbf{f}_{II}$ – Second Order Forced-Forced Frequency Matrix
$\mathbf{F}_{ij}$ **-** Contribution Vector of **F** from Player *i* to *j*
$\mathbf{f}_{IP}$ – Second Order Forced-Planned Frequency Matrix
FIPC – Vector of Outage Frequency of Control Points
$\mathbf{f}_P$ – Intrinsic Planned Outage Frequency Vector
$\mathbf{f}_{PI}$ – Second Order Planned-Forced Frequency Matrix
$\mathbf{f}_{Pj}$ – Sub Vector of $\mathbf{f}_P$ from Player *j*
$\mathbf{f}_{PD}$ – Diagonal Matrix of $\mathbf{f}_P$
$\mathbf{F}_P$ – Total Planned Outage Frequency Vector
$\mathbf{F}_{Pij}$ – Contribution Vector of $\mathbf{F}_P$ from Player *i* to *j*
$\mathbf{f}_{PP}$ – Second Order Planned-Planned Frequency Matrix
$\mathbf{F}_R$ **–** Vector of Protection Refusal Frequency
I – Forced Adjacency Matrix
i – Integer number less then *n*
j – Integer number less then *n*
K – Outage Duration Limit Vector
$\mathbf{K}_D$ – Outage Duration Limit Diagonal Matrix
l – Intrinsic Connected Load Vector
$\mathbf{l}_D$ –Diagonal Matrix of **l**
LOLE – Vector of Loss of Load Expectation
LOLP – Vector of Loss of Load Probability
n – Integer number of grid elements
N – Integer number of grid players
P – Planned Adjacency Matrix
$\mathbf{P}_C$ – Matrix of Grid Control Points
r – Vector of Intrinsic Time to Restore
$\mathbf{r}_D$ – Diagonal Matrix of **r**
r – Integer number less then *n*
R – Risk Vector
$\mathbf{R}_{AI}$ – Total Time to Restore after a Forced Outage
$\mathbf{R}_{AP}$ – Total Time to Restore after a Planned Outage
T – Time Span of Analysis
T – Protective Adjacency Matrix
$\mathbf{T}_C$ – Protective Vulnerability Matrix
$\mathbf{T}_{Cij}$ – Protective Vulnerability of Player *i* from *j*
V – Intrinsic Protective Vulnerability Vector
$\mathbf{V}_D$ – Diagonal Matrix of **V**

3. Component Modeling

Any components on a power grid has a behavior that depends on its intrinsic reliability, planned outages, protection, operation and connected load. These aspects can be modeled by defining vectors and diagonal matrices for their Intrinsic Forced Outage Frequency ($\mathbf{f}_I$), Intrinsic Planned Outage Frequency ($\mathbf{f}_P$), Intrinsic Protection Reliability **(C)**, Intrinsic Protective Vulnerability Vector **(V)**, Intrinsic Time to Restore (**r**), Intrinsic Forced Outage Duration ($\mathbf{d}_I$), Intrinsic Planned Outage Duration ($\mathbf{d}_P$), and Intrinsic Connected Load (**l**), partitioned by the *N* connected agents on the grid:

$$\mathbf{f}_I = \begin{bmatrix} f_{I1} & f_{I2} & \ldots & f_{In} \end{bmatrix}^T, \qquad \mathbf{f}_{ID} = diag[\mathbf{f}_I],$$
$$\mathbf{f}_P = \begin{bmatrix} f_{P1} & f_{P2} & \ldots & f_{Pn} \end{bmatrix}^T, \qquad \mathbf{f}_{PD} = diag[\mathbf{f}_P],$$
$$\mathbf{C} = \begin{bmatrix} C_1 & C_2 & \ldots & C_n \end{bmatrix}^T, \qquad \mathbf{C}_D = diag[\mathbf{C}],$$
$$\mathbf{V} = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix}^T, \qquad \mathbf{V}_D = diag[\mathbf{V}],$$
$$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \ldots & r_n \end{bmatrix}^T, \qquad \mathbf{r}_D = diag[\mathbf{r}],$$
$$\mathbf{d}_I = \begin{bmatrix} d_{I1} & d_{I2} & \ldots & d_{In} \end{bmatrix}^T, \qquad \mathbf{d}_{ID} = diag[\mathbf{d}_I],$$
$$\mathbf{d}_P = \begin{bmatrix} d_{P1} & d_{P2} & \ldots & d_{Pn} \end{bmatrix}^T, \qquad \mathbf{d}_{PD} = diag[\mathbf{d}_P],$$
$$\mathbf{l} = \begin{bmatrix} l_1 & l_2 & \ldots & l_n \end{bmatrix}^T, \qquad \mathbf{l}_D = diag[\mathbf{l}],$$

where *n* is the number of grid components, *diag* is a Matlab® function to construct/extract the diagonal of a matrix, and f_{Ii}, f_{Pi}, d_{Ii} e d_{Pi} represent the intrinsic mean frequencies and duration of each forced and planned component outage, including associated breakers, r_i is the manual or automatic reclosing time after each outage, and l_i is the direct external load connected to the item. C_i and V_i represent the probability that the protection of item *i* fails or trips unnecessarily, at a given instant, during faults on the reaching zone. Breakers can also be included on vectors **C** and **V**, with their probabilities of correct (when demanded) and incorrect (when not demanded) operation, respectively. The probability of being out of service will be obviously given by $1\text{-}C_i$, for both. In interconnected systems, with *N* players, $\mathbf{f}_{Ii}$, $\mathbf{f}_{Pi}$, $\mathbf{d}_{Ii}$, $\mathbf{d}_{Pi}$, $\mathbf{l}_i$ and $\mathbf{r}_i$ will be sub vectors associated to the equipments of player *i*. All forced outages, due to natural phenomena, environment, accidents, faults on equipments or on their protection and control systems, are included, as well as all planned outages, to maintenance, regulation, comissioning, modifications and refurbishing.

All second order contingencies, like forced or planned outages of item i while item j is out, are usually ignored, due to their low probabilities. However, they can be modeled by adding a fictitious item *ij* to vectors $\mathbf{f}_I$ and $\mathbf{f}_P$, whose frequency of forced or planned outage is given by the *ij* element of one of the square matrices:

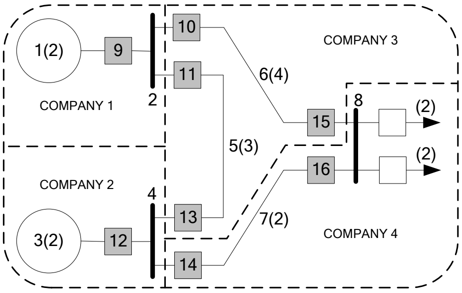
$$\mathbf{f}_{IP} = \mathbf{f}_I.diag[\mathbf{f}_{PD}\mathbf{d}_{PD}], \qquad \mathbf{f}_{II} = \mathbf{f}_I.diag[\mathbf{f}_{ID}\mathbf{d}_{ID}],$$
$$\mathbf{f}_{PI} = \mathbf{f}_P.diag[\mathbf{f}_{ID}\mathbf{d}_{ID}], \qquad \mathbf{f}_{PP} = \mathbf{f}_P.diag[\mathbf{f}_{PD}\mathbf{d}_{PD}].$$

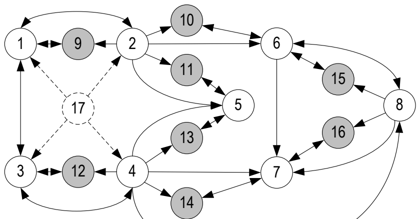
These matrices avail the frequency of a planned (subscript P) or forced (subscript I) event on element *i*, pondered by the probability of an impending event (planned or forced) originated on element *j*. Similar elements can be added to all other vectors to model duration, loads and protection reliability for these contingencies.

4. Topology Modeling

Operational dependency among equipments in a power grid can be modeled by an adjacency matrix that connects those items whose forced outages are related. The concept of forced adjacency applies not only to items located on the same protection zone, but also on distinct zones tripped by overload, faults, under and over voltages, or remote zones tripped by load or generation sharing schemes. A Forced Adjacency Matrix **I** can be defined by the expression and associated graph of Fig. 1, shown for a typical power grid, whose item capacities are indicated in parentheses. Generation companies 1 and 2, and transmission companies 3 and 4, are shown separated by dotted lines. The graph models all functional dependencies during forced outages of related components, obtained by contingency studies. Common failure modes can be modeled by fictitious nodes (shown dotted on the graph), such as a common failure on auxiliary services of companies 1 and 2, relating the affected items. Assume that all generators (4 pu) are necessary to attend the loads on bus 8, and line 6 (4 pu), will overload line 7 (2 pu) if it trips unexpectedly, but not if planned. Breakers are shaded, on the drawing and graph, to illustrate their inclusion in the model, but are not included on the matrix, to simplify the example.

$$\mathbf{I} = \left[I_{ij}\right] = p_i I p_j = \begin{cases} 1, & \text{if } p_i \text{ forces an outage of } p_j \text{ when } p_i \text{ trips automatically;} \\ 0, & \text{otherwise, where } p_i \text{ and } p_j \text{ are any items from the grid, and } i, j \le n. \end{cases}$$





$$\mathbf{I} = \begin{array}{cccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ \hline 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 8 \end{array}$$

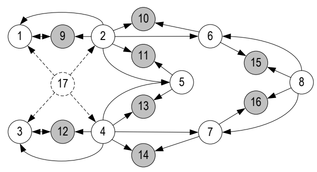
Figure 1 – Graph and Forced Adjacency Matrix of Typical Electrical Power Grid

Again, second order contingencies like a forced trip of an item *i* while item *j* is already out, can be modeled by adding a fictitious node *ij*, whose incidence elements on matrix **I** correspond to new items tripped in addition to those already out on the first contingency.

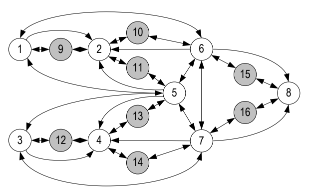
Likewise, it is possible to model the operational dependency among items by relating those whose planned outage will always occur at the same time. It applies, for instance, to transmission lines and transformers with their breakers, components that overload with the outage of other elements, items on the same protection zone, distinct items tripped to avoid overloads, or remote items to avoid operation of load sharing schemes, items on radial systems, etc. A Planned (Outage) Adjacency Matrix **P** can also be defined by the expression and graph of Fig. 2.

$$\mathbf{P} = \left[p_{ij}\right] = p_i P p_j = \begin{cases} 1, & \text{if } p_i \text{ impedes the operation of } p_j \text{ when } p_i \text{ is unavailable;} \\ 0, & \text{otherwise, where } p_i \text{ and } p_j \text{ are any items of a grid, and } i, j \le n. \end{cases}$$

$$\mathbf{T} = \left[t_{ij}\right] = p_i T p_j = \begin{cases} 1, & \text{If the protection or action of } p_i \text{ protects or senses faults on } p_j \text{, with } i \ne j; \\ 0, & \text{otherwise, where } p_i \text{ and } p_j \text{ are any items of a grid, and } i, j \le n. \end{cases}$$



$$\mathbf{P} = \begin{array}{cccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 8 \end{array}$$



$$\mathbf{T} = \begin{array}{cccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ \hline 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 5 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{array}$$

Figure 2 – Graphs of Planned and Protective Adjacency Matrices

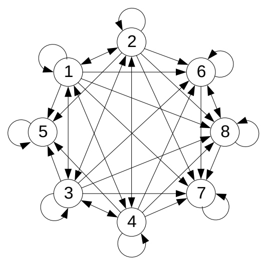
Items whose outage, by convenience, but not by necessity, are planned together, can be modeled by fictitious nodes (dotted on the graph), linked to the additional affected items. Similar nodes are added to model higher order planned outage events while other items are already out. The more meshed the grid, the sparser are matrices **P** and **I**. Fig. 2 shows this matrix (without the breakers) and the graph for the example power grid.

In the same way, the operational dependency among protection systems, breakers and protected components can be modeled relating those items whose faults are detected by each protection or affected by breaker trips. It applies, for instance, to items located on the same protection zone, or on adjacent zones at the reach of the protection, when it acts as a backup protection. It is possible to define, then, a Protective Adjacency Matrix **T**, by the expression and associated graph, shown on Fig. 2. Fictitious items corresponding to higher order contingencies should be modeled by adding adjacency cells to those items protected just in this case, but not by individual contingencies. The associated row and column are usually nulls.

These three matrices, **P**, **I** e **T**, model the grid topology. To cascade the reach of every outage it is necessary to derive the <u>Forced and Planned Reachability Matrices</u> ($\mathbf{A}_I$ e $\mathbf{A}_P$), and the <u>Protective Vulnerability Matrix</u> ($\mathbf{T}_C$) by the following operations (Boolean for $\mathbf{A}_I$ and $\mathbf{A}_P$, and algebraic for $\mathbf{T}_C$):

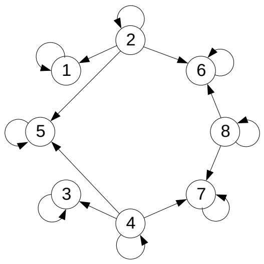
$$\mathbf{A}_I = [\mathbf{A}_{Iij}] = (\mathbf{I}+\mathbf{U})^r = (\mathbf{I}+\mathbf{U})^{r-1} \neq (\mathbf{I}+\mathbf{U})^{r-2},$$
$$\mathbf{A}_P = [\mathbf{A}_{Pij}] = (\mathbf{P}+\mathbf{U})^r = (\mathbf{P}+\mathbf{U})^{r-1} \neq (\mathbf{P}+\mathbf{U})^{r-2},$$
$$\mathbf{T}_C = [\mathbf{T}_{Cij}] = \mathbf{C}_D(\mathbf{T}-\mathbf{C}_D.\mathbf{T})^T + \mathbf{V}_D\mathbf{T},$$

where r (the smallest positive integer that satisfies the above equations) is the maximum extension of (forced or planned) cascading outages originated from any grid item, and **U** is the unit diagonal matrix. They link all items that must be tripped together, following the outage of one of them. In the third expression, ($\mathbf{T}$-$\mathbf{C}_D\mathbf{T}$) e ($\mathbf{V}_D\mathbf{T}$) are stochastic matrices of refusal or wrong trip chances from protection of component i, or breaker, for a fault in j. They give the probability of tripping of each grid element for a protection or breaker failure in other item, as a function of the <u>Protective Adjacency Matrix</u> **T.** Figure 3 shows the resultant graphs and reachability matrices for our typical electric grid, where all breakers were removed, to simplify the example.



$\mathbf{A}_I =$

	1	2	3	4	5	6	7	8	
	1	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1	2
	1	1	1	1	1	1	1	1	3
	1	1	1	1	1	1	1	1	4
	0	0	0	0	1	0	0	0	5
	0	0	0	0	0	1	1	1	6
	0	0	0	0	0	0	1	0	7
	0	0	0	0	0	1	1	1	8

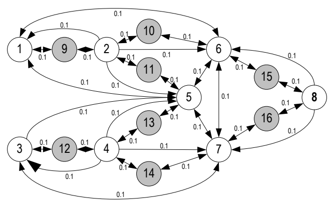


$\mathbf{A}_P =$

	1	2	3	4	5	6	7	8	
	1	0	0	0	0	0	0	0	1
	1	1	0	0	1	1	0	0	2
	0	0	1	0	0	0	0	0	3
	0	0	1	1	1	0	1	0	4
	0	0	0	0	1	0	0	0	5
	0	0	0	0	0	1	0	0	6
	0	0	0	0	0	0	1	0	7
	0	0	0	0	0	1	1	1	8

Figure 3 – Forced and Planned Outages Reachability Graphs and Matrices

Figure 4 illustrates the vulnerability graph and matrix for the grid example, with a constant protection reliability of 90%, and 1% of chance of protection refusal when demanded.



$\mathbf{T} =$

L1.	2.	3.	4.	5.	6.	7.	8.	
0	0	0	0	0.1	0.1	0	0	1
0.1	0	0	0	0.1	0.1	0	0	2
0	0	0	0	0.1	0	0.1	0	3
0	0	0.1	0	0.1	0	0.1	0	4
0.1	0	0.1	0	0	0.1	0.1	0	5
0.1	0	0	0	0	0	0.1	0	6
0	0	0.1	0	0	0.1	0	0	7
0	0	0	0	0	0.1	0.1	0	8

Figure 4 – Protection Vulnerability Graph and Matrix

In interconnected power systems, with N players, the elements of matrices $\mathbf{A_I}$**,** $\mathbf{A_P}$ e $\mathbf{T_C}$ can be partitioned by system (or company) in the <u>Interconnection Reachability Matrices</u> among players, $\mathbf{A}_{Iij}$, $\mathbf{A}_{Pij}$ e $\mathbf{T}_{Cij}$, which avail the impacts of forced and planned outages, including from protection or breaker failure, originated from player i over player j. Figures 1 to 4 show these partitioning among players 1 to 4, by dividing lines in the matrices. Matrices $\mathbf{A}_I$ e $\mathbf{A}_P$, cascade the consequences of each outage on grid topology. For logic consistency, $\mathbf{A}_I \geq \mathbf{A}_P$, always. The difference of these matrices, ($\mathbf{A}_I$-$\mathbf{A}_P$), defines all items that can be immediately reenergized, without waiting the restoration of the faulted item that originated the outage. If implemented in an energy management system, it could help on the system restoration after a major blackout. Matrix $\mathbf{A}_P$ also defines the items that must wait the restoration of a faulted component, before they can be returned to operation.

5. Interruption Frequencies

Taking these parameters, it is possible to avail the vectors and diagonal matrices of Total Forced and Planned Outage Frequencies ($\mathbf{F}_I$ and $\mathbf{F}_P$) and Total Outage Frequency from Protection Failure ($\mathbf{F}_C$) of all components, partitioned by N players:

$$\mathbf{F}_I = [F_{Ii}] = \mathbf{A}_I^T \mathbf{f}_I, \qquad \mathbf{F}_{ID} = diag(\mathbf{F}_I), \qquad i \le n,$$
$$\mathbf{F}_P = [F_{Pi}] = \mathbf{A}_P^T \mathbf{f}_P, \qquad \mathbf{F}_{PD} = diag(\mathbf{F}_P), \qquad i \le n,$$
$$\mathbf{F}_C = [F_{Ci}] = \mathbf{T}_C \mathbf{f}_I, \qquad \mathbf{F}_{CD} = diag(\mathbf{F}_C), \qquad i \le n,$$

giving the expected frequencies of forced and planned outage of each item, from intrinsic causes, or originated in other grid component or protection/breaker failure. Adding these parcels give the vector and diagonal matrix of Total Outage Frequency ($\mathbf{F}$) of each item, partitioned by N connected agents, and the Contribution Vector ($\mathbf{F}_{ij}$) from Player i to outages of player j:

$$\mathbf{F} = \mathbf{A}_P^T \mathbf{f}_P + [\mathbf{A}_I^T + \mathbf{T}_C]\mathbf{f}_I, \qquad \mathbf{F}_D = diag(\mathbf{F}),$$
$$\mathbf{F}_{ij} = \mathbf{F}_{Iij} + \mathbf{F}_{Pij} + \mathbf{F}_{Cij} = (\mathbf{A}_P^T)_{ij} \mathbf{f}_{Pj} + [(\mathbf{A}_I^T)_{ij} + \mathbf{T}_{Cij}]\mathbf{f}_{Ij}.$$

6. Interruption Duration

Mean interruption duration of each grid element results from the combined frequency and duration of forced and planned outages, and restoration times, measured by the vectors of Total Forced Outage Duration ($\mathbf{D}_{AI}$), Total Planned Outage Duration ($\mathbf{D}_{AP}$), Total Time to Restore after a Forced Outage ($\mathbf{R}_{AI}$), Total Time to Restore after a Planned Outage ($\mathbf{R}_{AP}$), and Total Outage Duration due to Protection Failure ($\mathbf{D}_{AC}$), partitioned by N players:

$$\mathbf{D}_{AI} = [D_{AIi}] = (\mathbf{d}_{ID} \mathbf{A}_P^T)\mathbf{f}_I, \qquad i \le n,$$
$$\mathbf{D}_{AP} = [D_{APi}] = (\mathbf{d}_{PD} \mathbf{A}_P^T)\mathbf{f}_P, \qquad i \le n,$$
$$\mathbf{R}_{AI} = [R_{AIi}] = (\mathbf{r}_D \mathbf{A}_I^T)\mathbf{f}_I, \qquad i \le n,$$
$$\mathbf{R}_{AP} = [R_{APi}] = (\mathbf{r}_D \mathbf{A}_P^T)\mathbf{f}_P, \qquad i \le n,$$
$$\mathbf{D}_{AC} = [D_{ACi}] = (\mathbf{r}_D \mathbf{T}_C)\mathbf{f}_I, \qquad i \le n.$$

The Total Outage Duration Vector ($\mathbf{D}_A$), results from the summation of these parcels, while the matrix of Total Forced Outage Duration of Player j due to i ($\mathbf{D}_{Aij}$) and the Mean Outage Duration Vector ($\mathbf{D}$) of each grid item, are obtained by multiplying $\mathbf{D}_A$ by the inverse Diagonal Matrix of $\mathbf{F}$ ($\mathbf{F}_D$):

$$\mathbf{D}_A = (\mathbf{d}_{ID}\mathbf{A}_P^T + \mathbf{r}_D \mathbf{A}_I^T + \mathbf{r}_D \mathbf{T}_C)\mathbf{f}_I + (\mathbf{d}_{PD} + \mathbf{r}_D)\mathbf{A}_P^T \mathbf{f}_P,$$
$$\mathbf{D}_{Aij} = [\mathbf{d}_{IDj}(\mathbf{A}_P^T)_{ij} + \mathbf{r}_{Dj}(\mathbf{A}_I^T)_{ij} + \mathbf{r}_{Dj}\mathbf{T}_{Cij}]\mathbf{f}_{Ij} + (\mathbf{d}_{PDj} + \mathbf{r}_{Dj})(\mathbf{A}_P^T)_{ij}\mathbf{f}_{Pj},$$
$$\mathbf{D} = \mathbf{D}_A . \mathbf{F}_D^{-1},$$

where $i,j \le N$ are any two grid players.

7. Control Points

On actual transmission and distribution systems, it is common to avail only some selected grid points, defined by a binary Matrix of Grid Control Points $\mathbf{P}_C$, where the non null items of its m x n elements define the m Grid Control Points of interest. Figure 4 shows the matrix $\mathbf{P}_C$ for the three buses (2, 4 and 8) of our example grid.

$$\mathbf{P}_C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_I^{''} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_P^{''} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{''} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 4 – Control Points and Restricted Reachability Matrices

Vectors of Outage Frequency (**FIPC**), Duration (**DIPC**), including Mean Duration (**DIPCM**) and Maximum Duration (**DMIPC**) of Control Points are derived by:

$$\mathbf{FIPC} = \mathbf{P}_C\mathbf{F},$$

$$\mathbf{DIPC} = \mathbf{P}_C\mathbf{D}_A,$$

$$\mathbf{DIPCM} = \mathbf{P}_C\mathbf{D},$$

$$\mathbf{DMIPC} = \mathbf{P}_C\, rowmax(horzcat(\mathbf{d}_{ID}\mathbf{A}_P^T + \mathbf{r}_D\mathbf{A}_I^T + \mathbf{r}_D\mathbf{T}_C, \mathbf{d}_{PD}\mathbf{A}_P^T + \mathbf{r}_D\mathbf{A}_P^T)),$$

where the maximum values are extracted from the parcels that contribute to the value of **D,** using MatLab® *rowmax* and *horzcat* matrix functions.

Depending on regulating rules, control points interruptions will be computed only if they exceed some limit durations, defined by an Outage Duration Limit Vector (**K**) and diagonal matrix ($\mathbf{K}_D$):

$$\mathbf{K} = [K_i], \qquad \mathbf{K}_D = diag(\mathbf{K}), \qquad i \le n,$$

where K_i is the minimum outage time needed to be accounted for control point i (typically K_i = 1 minute). In this case, new reachability matrices must be defined to filter out those outages whose durations are inferior to these limits, by the matrix expressions:

$$\mathbf{A}_P^{'} = [\mathbf{A}_{Pij}^{'}] = (\mathbf{d}_{PD} + \mathbf{r}_D)\mathbf{A}_P > \mathbf{K}_D\mathbf{A}_P \qquad i, j \le N,$$

$$\mathbf{A}_I^{'} = [\mathbf{A}_{Iij}^{'}] = (\mathbf{d}_{ID}\mathbf{A}_P + \mathbf{r}_D\mathbf{A}_I) > \mathbf{K}_D\mathbf{A}_I \qquad i, j \le N,$$

$$\mathbf{T}^{'} = [\mathbf{T}_{ij}^{'}] = \mathbf{r}_D\mathbf{T} > \mathbf{K}_D\mathbf{T} \qquad i, j \le N,$$

where use is made of the MatLab® matrix comparison operator, to nullify some original values. All previous expressions remain valid, with the above substitutions for reachability of Control Points.

8. Performance Indicators

Several other indicators can be evaluated to identify critical points on the grid, such as the expected vectors of Energy Not Supplied from Forced Outages ($\mathbf{ENS}_I$), Energy Not Supplied from Planned Outages ($\mathbf{ENS}_P$), Expected Mean Energy Not Supplied (**EENS**), Expected Demand Not Supplied (**EDNS**), Risk (**R**) and Protection Refusal Frequency ($\mathbf{F}_R$), for each component in a period, partitioned for N players:

$$\mathbf{ENS}_I = [ENS_{Ii}] = (\mathbf{r}_D\mathbf{A}_I + \mathbf{r}_D\mathbf{T}_C + \mathbf{d}_{ID}\mathbf{A}_P).\mathbf{l},$$
$$\mathbf{DNS}_I = \mathbf{f}_{ID}^{-1}.\mathbf{ENS}_I,$$
$$\mathbf{ENS}_P = [ENS_{Pi}] = [\mathbf{ENS}_{Pj}] = (\mathbf{r}_D + \mathbf{d}_{PD})\mathbf{A}_P\mathbf{l},$$
$$\mathbf{DNS}_P = [DNS_{Pi}] = [\mathbf{DNS}_{Pj}] = \mathbf{f}_{PD}^{-1}.\mathbf{ENS}_P,$$
$$\mathbf{ENS}_A = [ENS_{Ai}] = [\mathbf{ENS}_{Aj}] = \mathbf{l}_D\mathbf{D}_A,$$
$$\mathbf{EDNS} = [EDNS_i] = [\mathbf{EDNS}_j] = \mathbf{DNS}_I + \mathbf{DNS}_P,$$
$$\mathbf{EENS} = [EENS_i] = [\mathbf{EENS}_j] = T.\mathbf{EDNS},$$
$$\mathbf{R} = [R_i] = [\mathbf{R}_j] = (\mathbf{U} - \mathbf{C}_D)[\mathbf{1}^T(\mathbf{T} - \mathbf{C}_D\mathbf{T})],$$
$$\mathbf{F}_R = [F_{Ri}] = [\mathbf{F}_{Rj}] = \mathbf{R}\mathbf{f}_I^T,$$

where $i \le n$, $j \le N$, and $\mathbf{1} = [1,1,...1]^T$. Risk vectors **R** and $\mathbf{F}_R$ estimate the probability and frequency of occurrences of rare events with extreme losses, where all protection fail, including backup relays, during a fault.

The Expected Energy Not Supplied from Player *i* due to *j* ($EENS_{Aij}$), and the Total Expected Energy and Demand not Supplied ($EENS_{Ti}$, $EENS_T$ *e* $EDNS_T$), by each player and the total connected grid, and the Risk and Protection Refusal Frequency, are given by:

$$EENS_{Aij} = \mathbf{l}_{Di}\mathbf{D}_{Aij}, \qquad EENS_{Ti} = \mathbf{EENS}_{Ai}^T\mathbf{1}_i, \qquad i \le n,$$
$$EENS_{IT} = \mathbf{EENS}_{ITi}^T\mathbf{1}_i, \qquad EENS_{PT} = \mathbf{EENS}_{PTi}^T\mathbf{1}_i, \qquad i \le n$$
$$R_j = (\mathbf{R}_j^T\mathbf{f}_{Ij})/(\mathbf{f}_{Ij}^T\mathbf{1}_j), \qquad EENS_T = \mathbf{EENS}_T^T\mathbf{1}_T, \qquad i \le n,$$
$$EDNS_T = \mathbf{EDNS}^T\mathbf{1}_T, \qquad EDNS_{IT} = \mathbf{EDNS}_{ITi}^T\mathbf{1}_i, \qquad i \le n,$$
$$EDNS_{PT} = \mathbf{EDNS}_{PTi}^T\mathbf{1}_i, \qquad F_{Rj} = \mathbf{R}_j^T\mathbf{f}_{Ij}, \qquad i \le n,$$

where $\mathbf{1}_i$, $\mathbf{1}_T = [1,1,...1]^T$, e $i,j \le N$.

The Loss of Load Probability and Expectation (**LOLP, LOLE)**, of all players ($\mathbf{LOLP}_S$, $\mathbf{LOLE}_S$) and the whole grid ($LOLP_S$, $LOLE_S$), are evaluated by defining new reachability matrices, restricted to those items with connected loads, using the MatLab® matrix comparison operator:

$$\mathbf{A}_I'' = (\mathbf{A}_I\mathbf{l}_D > \mathbf{0}), \qquad \mathbf{A}_{Ii}'' = (\mathbf{A}_{Ii}\mathbf{l}_{Di} > \mathbf{0}_i), \qquad i \le n,$$
$$\mathbf{A}_P'' = (\mathbf{A}_P\mathbf{l}_D > \mathbf{0}), \qquad \mathbf{A}_{Pi}'' = (\mathbf{A}_{Pi}\mathbf{l}_{Di} > \mathbf{0}_i), \qquad i \le n,$$
$$\mathbf{T}'' = (\mathbf{T}.\mathbf{l}_D > \mathbf{0}), \qquad \mathbf{T}_i'' = (\mathbf{T}_i\mathbf{l}_{Di} > \mathbf{0}_i), \qquad i \le n,$$
$$\mathbf{T}_C'' = \mathbf{C}_D(\mathbf{T}'' - \mathbf{C}_D.\mathbf{T}'')^T \quad \mathbf{T}_{Ci}'' = \mathbf{C}_{Di}(\mathbf{T}_i'' - \mathbf{C}_{Di}.\mathbf{T}_i'')^T, \qquad i \le n$$

where $\mathbf{0} = [0,0,...0]^T$. Summing all maximum duration interruptions, pondered by their originated event frequencies, gives an estimation of the fraction of time or probability of happening some load curtailment:

$$\mathbf{LOLP} = [LOLP_i] = (\mathbf{d}_{ID}\mathbf{A}_P''^T + \mathbf{r}_D\mathbf{A}_I''^T + \mathbf{r}_D\mathbf{T}_C'')\mathbf{f}_I + (\mathbf{d}_{PD} + \mathbf{r}_D)\mathbf{A}_P''^T\mathbf{f}_P,$$
$$\mathbf{LOLP}_S = \mathbf{f}_{Ii}^T rowmax(\mathbf{r}_{Di}\mathbf{A}_{Ii}'' + \mathbf{d}_{IDi}\mathbf{A}_{Pi}'' + \mathbf{r}_{Di}\mathbf{T}_{Ci}''^T) + \mathbf{f}_{Pi}^T rowmax((\mathbf{r}_{Di} + \mathbf{d}_{PDi})\mathbf{A}_{Pi}''),$$
$$LOLP_S = \mathbf{f}_I^T rowmax(\mathbf{r}_D\mathbf{A}_I'' + \mathbf{d}_{ID}\mathbf{A}_P'' + \mathbf{r}_D\mathbf{T}_C''^T) + \mathbf{f}_P^T rowmax((\mathbf{r}_D + \mathbf{d}_{PD})\mathbf{A}_P''),$$
$$\mathbf{LOLE} = [LOLE_i] = T.\mathbf{LOLP},$$
$$\mathbf{LOLE}_S = [LOLE_{Si}] = T.\mathbf{LOLP}_S,$$
$$LOLE_S = T.LOLP_S,$$

where T = time span of analysis.

Many similar indicators can be derived substituting the load demand of each node in vector **l**, by other variable of interest, such as the number of consumers, habitants, industrial production, income, revenue, social cost, etc., affected by an outage. Using the same expressions of **LOLP**, **DNS** e **ENS**, it is possible to estimate, for instance, the levels of (not) attainment of consumers, by indexes such as:

- CAIFI – Customer Average Interruption Frequency,
- CAIDI – Customer Average Interruption Duration, and
- LOCP – Loss of Customer Probability.

As such, they can be used, for example, in contractual negotiations, rate studies and loss compensations during outages on connected systems.

9. Conclusions

The following aspects distinguish the proposed method, in evaluating performance indexes and risk sharing among players, due to cascading disturbances:

(a) Evaluation of maintenance, operation and protection of power systems;
(b) Simulation of grid topology, with forced and planned functional dependencies;
(c) Graphical representation of functional dependencies by directed graphs;
(d) Modeling of protection reliability and reach;
(e) Representation of common mode failures and stuck breakers and protections;
(f) Optional modeling of higher order contingencies;
(g) Inclusion of remote causalities from teleprotections and from load and generation shedding;
(h) Use of traditionally available data from maintenance and operation;
(i) Evaluation of continuity, technical, economic and social indicators;
(j) Elicitation of the contribution and responsibilities of outages among connected companies; and
(k) Formalization by matrix algebra, with trivial implementation on computers.

BIBLIOGRAPHY

[1] Siqueira, I. P., "Optimum Reliability-Centered Maintenance Task Frequencies for Power System Equipments", 8th PMAPS, IEEE, 2004.
[2] Siqueira, I. P., "Measuring the Impact of an RCM Program on Power System Performance", IEEE PES General Meeting, IEEE, 2005.
[3] Billinton, R., Allan, R. N., "Reliability Evaluation of Engineering Systems", Pitman, 1983
[4] Endrenyi, J., "Reliability Modeling in Electric Power Systems", John Wiley & Sons, 1978.
[5] Sullivan, R. L., "Power System Planning", McGraw-Hill, 1977.
[6] Sage, A. P., "Methodology for Large-Scale Systems", McGraw-Hill, 1977.
[7] Li, W., "Risk Assessment of Power Systems – Models, Methods, and Applications", Wiley, 2004
[8] Siqueira, I.P., "Manutenção Centrada na Confiabilidade – Manual de Implementação", ISBN 85-7303-566-8, Editora QualityMark, Rio de Janeiro, Brazil, 2005.